Graphs and the Chromatic Polynomial

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A **complete graph** is maximally connected simple graph in which each vertex is connected to every other vertex.

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Read's Conjecture states that the chromatic polynomial of a graph has coefficients that are log concave, and hence unimodal.